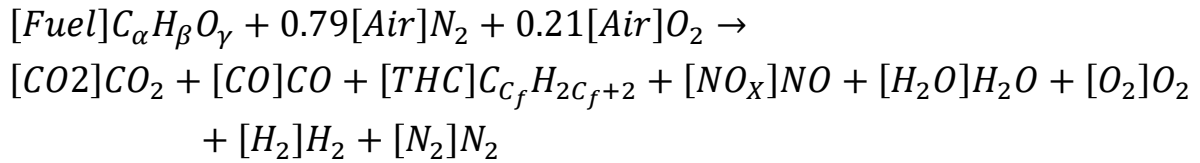


EGR and Lambda Factor Models from Measured Gas Concentrations



Carbon balance

$$\alpha[Fuel] = [CO_2] + [CO] + C_f[THC]$$

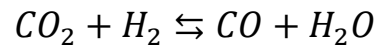
Oxygen balance

$$\gamma[Fuel] + 2 \times 0.21[Air] = 2[CO_2] + [CO] + [NO_x] + [H_2O] + 2[O_2]$$

Hydrogen balance

$$\beta[Fuel] = (2C_f + 2)[THC] + 2[H_2O] + 2[H_2]$$

CO/CO₂ equilibrium



$$K_{eq} = \frac{[CO][H_2O]}{[CO_2][H_2]} = 3.5 \text{ for } 1750K \text{ (mean burning temperature)}$$

Developing equations

Molecular Hydrogen concentration

$$[H_2] = \frac{\beta}{2}[Fuel] - [H_2O] - (C_f + 1)[THC]$$

Water concentration

$$[H_2O] = K_{eq} \frac{[CO_2]}{[CO]} [H_2] = K_{eq} \frac{[CO_2]}{[CO]} \left\{ \frac{\beta}{2}[Fuel] - [H_2O] - (C_f + 1)[THC] \right\}$$

[THC] is a small term, $O(10^{-5})$,

$$[H_2O] = \frac{\frac{\beta}{2}[Fuel]}{\left\{ 1 + \frac{[CO]}{K_{eq}[CO_2]} \right\}} = \frac{K_{eq} \frac{\beta}{2}[Fuel]}{\left\{ K_{eq} + \frac{[CO]}{[CO_2]} \right\}} = \frac{3.5 \frac{\beta}{2}[Fuel]}{\left\{ 3.5 + \frac{[CO]}{[CO_2]} \right\}}$$

Fuel concentration

$$[Fuel] = \frac{[CO_2] + [CO] + C_f[THC]}{\alpha}$$

Air concentration

$$0.21[Air] = \frac{1}{2} \times \{2[CO_2] + [CO] + [NO_x] + [H_2O] + 2[O_2]\} - \frac{\gamma}{2}[Fuel]$$

$$0.21[Air] = [CO_2] + \frac{[CO]}{2} + \frac{[NO_x]}{2} + \frac{1}{2} \times \frac{3.5 \frac{\beta}{2} [Fuel]}{\left\{3.5 + \frac{[CO]}{[CO_2]}\right\}} + [O_2] - \frac{\gamma}{2}[Fuel]$$

$$0.21[Air] = [CO_2] + \frac{[CO]}{2} + [O_2] + \frac{[NO_x]}{2} + \frac{3.5}{\left\{3.5 + \frac{[CO]}{[CO_2]}\right\}} \times \frac{\beta}{4} [Fuel] - \frac{\gamma}{2} [Fuel]$$

$$0.21[Air] = [CO_2] + \frac{[CO]}{2} + [O_2] + \frac{[NO_x]}{2} + \left\{ \frac{3.5}{\left\{3.5 + \frac{[CO]}{[CO_2]}\right\}} \times \frac{\beta}{4} - \frac{\gamma}{2} \right\} [Fuel]$$

$$0.21[Air] = [CO_2] + \frac{[CO]}{2} + [O_2] + \frac{[NO_x]}{2} + \left\{ \frac{3.5}{\left\{3.5 + \frac{[CO]}{[CO_2]}\right\}} \times \frac{\beta}{4} - \frac{\gamma}{2} \right\} \frac{[CO_2] + [CO] + C_f[THC]}{\alpha}$$

Defining,

$$O_{cv} = \frac{\gamma}{\alpha}$$

$$H_{cv} = \frac{\beta}{\alpha}$$

$$0.21[Air] = [CO_2] + \frac{[CO]}{2} + [O_2] + \frac{[NO_x]}{2} + \left\{ \frac{3.5}{\left\{3.5 + \frac{[CO]}{[CO_2]}\right\}} \times \frac{H_{cv}}{4} - \frac{O_{cv}}{2} \right\} \{[CO_2] + [CO] + C_f[THC]\}$$

[THC] is a small term, $O(10^{-5})$,

$$[Air] = \frac{1}{0.21} \left\{ [CO_2] + \frac{[CO]}{2} + [O_2] + \frac{[NO_x]}{2} + \left\{ \frac{3.5}{\left\{3.5 + \frac{[CO]}{[CO_2]}\right\}} \times \frac{H_{cv}}{4} - \frac{O_{cv}}{2} \right\} \{[CO_2] + [CO]\} \right\}$$

Air/Fuel ratio calculation (molar base)

$$\frac{[Air]}{[Fuel]} = \frac{\frac{\alpha}{0.21} \left\{ [CO_2] + \frac{[CO]}{2} + [O_2] + \frac{[NO_x]}{2} + \left\{ \frac{3.5}{\left\{3.5 + \frac{[CO]}{[CO_2]}\right\}} \times \frac{H_{cv}}{4} - \frac{O_{cv}}{2} \right\} \{[CO_2] + [CO]\} \right\}}{[CO_2] + [CO] + C_f[THC]}$$

For stoichiometric conditions (molar base),

$$[CO] = [O_2] = [NO_x] = [THC] = 0$$

$$\left. \frac{[Air]}{[Fuel]} \right|_{stoich} = \frac{\alpha}{0.21} \left(1 + \frac{H_{cv}}{4} - \frac{O_{cv}}{2} \right)$$

Air/Fuel ratio in mass base,

$$M_{air} = 28.84 \frac{kg}{kmol}$$

$$M_{fuel} = 12\alpha + \beta + 16\gamma \quad kg/kmol$$

$$AF = \frac{m_{air}}{m_{fuel}} = \frac{[Air]}{[Fuel]} \frac{M_{air}}{M_{fuel}}$$

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Brettschneider Equation, 1979, for Lambda factor:

$$\lambda = \frac{\frac{[Air]}{[Fuel]} \frac{M_{air}}{M_{fuel}}}{\frac{[Air]}{[Fuel]} \Big|_{stoich} \frac{M_{air}}{M_{fuel}}} = \frac{\frac{[Air]}{[Fuel]}}{\frac{[Air]}{[Fuel]} \Big|_{stoich}}$$

$$\lambda = \frac{\left\{ [CO_2] + \frac{[CO]}{2} + [O_2] + \frac{[NO_x]}{2} + \left\{ \frac{3.5}{3.5 + \frac{[CO]}{[CO_2]}} \times \frac{H_{cv}}{4} - \frac{O_{cv}}{2} \right\} \{ [CO_2] + [CO] \} \right\}}{\left(1 + \frac{H_{cv}}{4} - \frac{O_{cv}}{2} \right) \{ [CO_2] + [CO] + C_f[THC] \}}$$